Investigation of Moving-Bank Multiple Model Adaptive Algorithms

Peter S. Maybeck* and Karl P. Hentz†
Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio

The feasibility of a moving-bank multiple model adaptive estimator/controller is examined. A significant reduction in the number of required elemental filters is accomplished through a dynamic redeclaration of the positions that the elemental filters occupy in parameter space (as compared to a conventional full-bank multiple model adaptive algorithm). Critical to the performance of the moving-bank estimator is the decision method that governs movement of the bank of elemental filters. Five such methods are developed, and their performances are compared to each other and to a benchmark filter with artificial knowledge of the "true" parameter value. Three adaptive controller algorithms are also generated and evaluated. A simple but physically motivated example is used to gain insights into the relative performance potentials of the proposed algorithms.

Introduction

In many estimation and control problems, uncertain parameters exist within the system model used for algorithm design (i.e., of Kalman filters, regulators, tracking algorithms, etc.). For instance, in the design of controllers for flexible vehicles, parameters that describe bending modes are often subject to considerable uncertainty. Similarly in tracking problems, the target characteristics cannot be specified *a priori* with absolute assurance, and some, such as rms acceleration level, change continuously in real time. In other applications, such as failure detection, parameters can undergo large jump changes.

Such problems give rise to the need for the estimation of parameter values simultaneously with estimation (and control) of state variables. One means of accomplishing this in a manner that is ideally suited to distributed computation is multiple model adaptive estimation (MMAE).[1-6] The system is assumed to be adequately represented by a linear stochastic state model, with uncertain parameters affecting the matrices defining the structure of the model or depicting the statistics of the noises entering it. It is further assumed that the parameters can take on only discrete values; either this is physically reasonable (as for failure detection) or representative discrete values are chosen throughout the continuous range of possible values. A Kalman filter is then designed for each choice of parameter value, resulting in a bank of K separate "elemental" filters. Based on the observed characteristics of the residuals in these K filters, the conditional probabilities of each discrete parameter value being "correct" (given the measurement history to that time) are evaluated iteratively. The state estimate of each filter is weighted by its corresponding probability, and the adaptive state estimate is produced as the probability-weighted average of the elemental filter outputs. As an alternative (using maximum a posteriori, or MAP, rather than minimum mean square error, or MMSE, criteria for optimality), the state estimate from the single elemental filter associated with the highest conditional probability can be selected as the output of the adaptive state estimator.

For control applications, the state estimate so generated can be premultiplied by a controller gain established via forced certainty equivalence design,[7,8] producing MMAE-based control. This controller gain may itself be evaluated using the parameter estimate, or it may be based on a single nominal parameter value. Another possible adaptive controller structure is the multiple model adaptive controller (MMAC),[7,9] in which a separate controller gain is associated with each elemental filter in the bank. Here the control is produced as the probability-weighted average of elemental controller outputs (or the control from the single elemental controller corresponding to the highest conditional probability of being based on the "true" parameter).

Initial development and investigations of multiple model algorithms assumed uncertain but constant parameters, and some useful convergence results have been obtained for this class of problems.[10-12] For the case of time-varying parameters, one ad hoc approach has been to use an algorithm designed under the assumption of constant parameters, but providing a lower bound for the computed probabilities to prevent the algorithm from "locking onto" a single parameter value.[2,4] Another approach has been to match each elemental filter to a time history of parameter values rather than just one constant value.[4-6,8,13] This would require K^i elemental filters at sample time t_i, which would be impractical for actual implementation. Various approaches have been taken to reduce the computational burden of the algorithm, including the use of Markov models for parameter variation,[4,13,14] "pruning" and "merging" of branches in a "tree" of possible parameter time histories,[5,6,8,15] hierarchical structuring,[16] and dynamic coarse-to-finer rediscretization.[17]

Multiple model adaptation has been successfully applied to a number of practical problems. It has exhibited promising results in the tracking of maneuvering targets,[18-26] and has also been used in flight control,[9] multiple hypothesis testing,[5,6,8,27] detection of incidents on freeways,[28] adaptive deconvolution of seismic signals,[29] and problems where initial uncertainties are so large that nonadaptive extended Kalman filters diverge.[30,31]

One basic problem with this approach is the number of filters (and controllers) in the bank. For instance, if there are two uncertain parameters and each can assume ten possible values, then $10^2 = 100$ separate filters (and controllers) must be implemented, even if parameters are treated as unknown *constants*. To circumvent this problem, one can think of implementing a "moving bank" of fewer estimators.[32,33] In the previous example, one might choose the three discrete values of each parameter that most closely surround the

Received Jan. 13, 1986; revision received July 21, 1986.

*Professor, Department of Electrical and Computer Engineering. Member AIAA.

†Captain, USAF, Department of Electrical and Computer Engineering; currently with the Air Force Weapons Laboratory, Kirtland AFB, NM.

estimated value, thus requiring only $3^2 = 9$ separate elemental filters. The particular choice of nine filters would then depend on the most recent estimate of the parameters, generated in real time.

Maintaining fewer elemental filters in the bank enhances the feasibility of multiple model algorithms, but it could aggravate the behavior of making hasty decisions when the "true model" is not included in the filter's model set, as was observed earlier.[9] Some research has been directed at the information theoretic problem associated with this condition.[34-37] Thus, it is essential to demonstrate that any proposed decision logic for moving and changing the size of the bank yields effective performance under this and other realistic conditions.

This paper develops the decision logic for a moving-bank multiple model algorithm and evaluates its potential. The subsequent section presents the structure of a multiple model adaptive estimator; five possible logics for deciding to move or change the size of the moving bank are then developed. Adaptive control based on these ideas is developed and, finally, the performance capabilities of the proposed algorithms are assessed.

Multiple Model Adaptive Estimation

Let $\boldsymbol{a}$ denote the vector of uncertain parameters in a given linear stochastic state model for a dynamic system. These parameters can affect the matrices defining the structure of the model or depicting the statistics of the noises entering it. In order to make simultaneous estimation of states and parameters tractable, the continuous range of $\boldsymbol{a}$ values is discretized into K representative values. If we define the hypothesis conditional probability $p_k(t_i)$ as the probability that $\boldsymbol{a}$ assumes the value $\boldsymbol{a}_k$ (for $k = 1, 2, \cdots, K$), conditioned on the observed measurement history to time t_i

$$p_k(t_i) = \text{Prob}\{\boldsymbol{a} = \boldsymbol{a}_k | \boldsymbol{Z}(t_i) = \boldsymbol{Z}_i\} \tag{1}$$

then it can be shown[1-4] that $p_k(t_i)$ can be evaluated recursively for all k via the iteration

$$p_k(t_i) = \frac{f_{z(t_i)|\boldsymbol{a},\boldsymbol{Z}(t_{i-1})}(\boldsymbol{z}_i|\boldsymbol{a}_k, \boldsymbol{Z}_{i-1}) \cdot p_k(t_{i-1})}{\sum_{j=1}^{K} f_{z(t_i)|\boldsymbol{a},\boldsymbol{Z}(t_{i-1})}(\boldsymbol{z}_i|\boldsymbol{a}_j, \boldsymbol{Z}_{i-1}) \cdot p_j(t_{i-1})} \tag{2}$$

in terms of the previous values of $p_1(t_{i-1}), \cdots, p_K(t_{i-1})$, and conditional densities for the current measurement $\boldsymbol{z}(t_i)$ to be explicitly defined in Eq. (12). Notationally, the measurement history random vector $\boldsymbol{Z}(t_i)$ is made up of partitions $\boldsymbol{z}(t_1), \cdots, \boldsymbol{z}(t_i)$ that are the measurement vectors available at the sample times $t_1, \cdots, t_i$; similarly, the realization $\boldsymbol{Z}_i$ of the measurement history vector has partitions $\boldsymbol{z}_1, \cdots, \boldsymbol{z}_i$. Furthermore, the Bayesian minimum mean square error estimate of the state is the probability-weighted average

$$\hat{\boldsymbol{x}}(t_i^+) = E\{\boldsymbol{x}(t_i)|\boldsymbol{Z}(t_i) = \boldsymbol{Z}_i\} = \sum_{k=1}^{K} \hat{\boldsymbol{x}}_k(t_i^+) \cdot p_k(t_i) \tag{3}$$

where $\hat{\boldsymbol{x}}_k(t_i^+)$ is the state estimate generated by a Kalman filter based on the assumption that the parameter vector equals $\boldsymbol{a}_k$. More explicitly, let the model corresponding to $\boldsymbol{a}_k$ be described by an "equivalent discrete-time model"[4,7] for a continuous-time system with sampled data measurements

$$\boldsymbol{x}_k(t_{i+1}) = \boldsymbol{\Phi}_k(t_{i+1}, t_i)\boldsymbol{x}_k(t_i) + \boldsymbol{B}_k(t_i)\boldsymbol{u}(t_i) + \boldsymbol{G}_k(t_i)\boldsymbol{w}_k(t_i) \tag{4}$$

$$\boldsymbol{z}(t_i) = \boldsymbol{H}_k(t_i)\boldsymbol{x}_k(t_i) + \boldsymbol{v}_k(t_i) \tag{5}$$

where $\boldsymbol{x}_k$ is the state, $\boldsymbol{u}$ is a control input, $\boldsymbol{w}_k$ is discrete-time zero-mean white Gaussian dynamics noise of covariance $\boldsymbol{Q}_k(t_i)$ at each t_i, $\boldsymbol{z}$ is the measurement vector, and $\boldsymbol{v}_k$ is discrete-time zero-mean white Gaussian measurement noise of covariance $\boldsymbol{R}_k(t_i)$ at t_i, assumed independent of $\boldsymbol{w}_k$; $\boldsymbol{x}(t_o)$ is modeled as Gaussian, with mean $\hat{\boldsymbol{x}}_{ko}$ and covariance $\boldsymbol{P}_{ko}$ and is assumed independent of $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$. Based on this model, the Kalman filter is specified by the measurement update

$$\boldsymbol{A}_k(t_i) = \boldsymbol{H}_k(t_i)\boldsymbol{P}_k(t_i^-)\boldsymbol{H}_k^T(t_i) + \boldsymbol{R}_k(t_i) \tag{6}$$

$$\boldsymbol{K}_k(t_i) = \boldsymbol{P}_k(t_i^-)\boldsymbol{H}_k^T(t_i)\boldsymbol{A}_k^{-1}(t_i) \tag{7}$$

$$\hat{\boldsymbol{x}}_k(t_i^+) = \hat{\boldsymbol{x}}_k(t_i^-) + \boldsymbol{K}_k(t_i)[\boldsymbol{z}_i - \boldsymbol{H}_k(t_i)\hat{\boldsymbol{x}}_k(t_i^-)] \tag{8}$$

$$\boldsymbol{P}_k(t_i^+) = \boldsymbol{P}_k(t_i^-) - \boldsymbol{K}_k(t_i)\boldsymbol{H}_k(t_i)\boldsymbol{P}_k(t_i^-) \tag{9}$$

and the propagation relation

$$\hat{\boldsymbol{x}}_k(t_{i+1}^-) = \boldsymbol{\Phi}_k(t_{i+1}, t_i)\hat{\boldsymbol{x}}_k(t_i^+) + \boldsymbol{B}_k(t_i)\boldsymbol{u}(t_i) \tag{10}$$

$$\boldsymbol{P}_k(t_{i+1}^-) = \boldsymbol{\Phi}_k(t_{i+1}, t_i)\boldsymbol{P}_k(t_i^+)\boldsymbol{\Phi}_k^T(t_{i+1}, t_i) + \boldsymbol{G}_k(t_i)\boldsymbol{Q}_k(t_i)\boldsymbol{G}_k^T(t_i) \tag{11}$$

Thus, the multiple model adaptive filtering algorithm is composed of a bank of K separate Kalman filters, each based on a particular value $\boldsymbol{a}_1, \cdots, \boldsymbol{a}_K$ of the parameter vector, as depicted in Fig. 1. When the measurement $\boldsymbol{z}_i$ becomes available at t_i, the residuals $\boldsymbol{r}_1(t_i), \cdots, \boldsymbol{r}_K(t_i)$ are generated in the K filters, as shown in Eq. (8), and used to compute $p_1(t_i), \cdots, p_K(t_i)$ via Eq. (2). Each numerator density function in Eq. (2) is given by

$$f_{z(t_i)|\boldsymbol{a},\boldsymbol{Z}(t_{i-1})}(\boldsymbol{z}_i|\boldsymbol{a}_k, \boldsymbol{Z}_{i-1}) = \frac{1}{(2\pi)^{m/2}|\boldsymbol{A}_k(t_i)|^{\frac{1}{2}}}\exp\{\cdot\}$$

$$\{\cdot\} = \{-\tfrac{1}{2}\boldsymbol{r}_k^T(t_i)\boldsymbol{A}_k^{-1}(t_i)\boldsymbol{r}_k(t_i)\} \tag{12}$$

where m is the measurement dimension and $\boldsymbol{A}_k(t_i)$ is calculated in the kth Kalman filter, as in Eq. (6). The denominator in Eq. (2) is simply the sum of all the computed numerator terms and thus is the scale factor required to ensure that the $p_k(t_i)$'s sum to one.

One expects the residuals of the Kalman filter based upon the "best" model to have mean squared value most in consonance with its own computed $\boldsymbol{A}_k(t_i)$, while "mismatched"

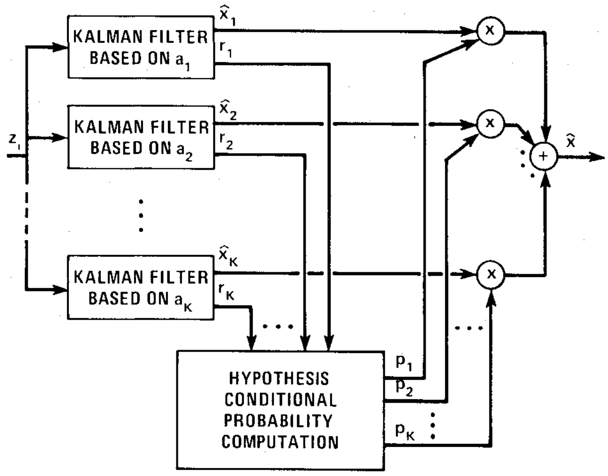

Fig. 1 Multiple model adaptive estimation algorithm.

filters will have larger residuals than anticipated through $A_k(t_i)$. Therefore, Eqs. (2), (3), and (6)-(12) will most heavily weight the filter based upon the most correct assumed parameter value. However, the performance of the algorithm depends on there being significant differences in the characteristics of residuals in "correct" vs. "mismatched" filters. Each filter should be tuned for the best performance when the "true" values of the uncertain parameters are identical to its assumed value for these parameters. One should specifically avoid the "conservative" philosophy of adding considerable dynamics pseudonoise, often used to open the bandwidth of a single Kalman filter to guard against divergence, since this tends to mask the differences between good and bad models.

The moving-bank multiple model adaptive estimator is identical to the full-bank estimator just described, except that K does not correspond to the total number of possible parameter vector values. Instead, it is the smaller number of elemental Kalman filters maintained within the bank. Which particular K filters are in the bank at a given time is determined by one of the decision mechanisms of the next section.

Moving the Bank

When the "true" parameter point lies within the region of parameter space bracketed by the moving bank, the moving-bank estimator behaves much as the standard full-bank multiple model adaptive estimator. If the "true" parameter value should lie outside that region, or even inside the region but near its boundary, we must first detect this condition and then take some appropriate action, i.e., move or expand the bank in some fashion. Five decision logics were proposed for investigation.

Residual Monitoring

Let the likelihood quotient $L_k(t_i)$ be the quadratic form that appears in Eq. (12).

$$L_k(t_i) = r_k^T(t_i) A_k^{-1}(t_i) r_k(t_i) \tag{13}$$

In the case of scalar measurements, this is the current residual squared, divided by the filter-computed variance for the residual. When the true parameter value does not lie within the moving-bank region, all K likelihood quotients can be expected to exceed a threshold level T, the numerical value of which is set in an ad hoc manner during performance evaluations. Thus, a possible detection logic would indicate that the bank should be moved at time t_i if

$$\min\{L_1(t_i), L_2(t_i), \cdots, L_K(t_i)\} \geq T \tag{14}$$

Moreover, the elemental filter based on a_k nearest to the true parameter value should have the smallest likelihood quotient, thereby giving an indication of the direction to move the bank. While this logic would effectively respond to a real need to move the bank, it would also be prone to false alarms induced by single large samples of measurement noise.

Parameter Position Estimate Monitoring

Another means of keeping the true parameter value within the region bracketed by the moving bank is to keep the bank centered (as closely as possible, in view of the discrete values a is allowed to assume) on the current estimate of the parameter. This estimate is[4]

$$\hat{a}(t_i) = E\{a | Z(t_i) = Z_i\} = \sum_{k=1}^{K} a_k \cdot p_k(t_i) \tag{15}$$

If the distance from the parameter value associated with the center of the bank to $\hat{a}(t_i)$ becomes larger than some chosen threshold, a move of the bank in that direction is indicated. Since $\hat{a}(t_i)$ depends on a *history* of measurements, it is less prone to the false alarms discussed in the previous paragraph.

Parameter Position and "Velocity" Estimate Monitoring

If the true parameters are slowly varying, past values of $\hat{a}(t_j)$ can be used to generate an estimate of parameter "velocity." This, along with the current position estimate $\hat{a}(t_i)$, can be used to compute a predicted parameter position, one sample period into the future. If the distance between the bank center and that projection exceeds some selected threshold, the bank can be moved in anticipation of the true parameter movement.

Probability Monitoring

The conditional hypothesis probabilities $p_k(t_i)$ computed via Eq. (2) are another indication of the correctness of the parameter values a_k. If any of these rise above a chosen threshold level, the bank can be moved in the direction of the a_k associated with the highest $p_k(t_i)$. In this scheme, the bank seeks to center itself on the elemental filter with the highest conditional probability weighting. Again, since $p_k(t_i)$ depends on a history of measurements, this method should not be sensitive to single bad samples of measurement corruption, as discussed in the residual monitoring paragraph.

Parameter Estimation Error Covariance Monitoring

This concept is discussed last because it has a somewhat different purpose than deciding when to move the bank and in what direction. It is also possible to change the *size* of the bank by altering the discretization level in parameter space. For example, initial acquisition may be enhanced by choosing the values $a_1, \cdots, a_K$ so that they coarsely encompass all possible parameter values instead of using a smaller sized bank and forcing it to seek a true parameter value that may well be outside the region of its assumed parameter values. Then, once a "good" parameter estimate had been achieved with this coarse discretization, the size of the bank could be contracted and the smaller bank could be centered on that "good" value. To help make such a contraction decision, it would be useful to monitor the parameter estimation error conditional covariance, computable[4] as

$$P_a(t_i) = E\{[a - \hat{a}(t_i)][a - \hat{a}(t_i)]^T | Z(t_i) = Z_i\}$$
$$= \sum_{k=1}^{K} [a_k - \hat{a}(t_i)][a_k - \hat{a}(t_i)]^T \cdot p_k(t_i) \tag{16}$$

When an appropriately chosen scalar function (norm) of this matrix falls below a selected threshold, the bank can be contracted about the parameter estimate. In this research, a weighted sum of diagonal terms is used as the matrix norm, and the moving bank is constrained to be a square region in the two-dimensional parameter space. One can also use different discretization coarseness decisions in individual directions of this space, allowing rectangular banks instead of only squares, with some benefit in performance.

An indication to expand the size of the bank can be obtained from residual monitoring: if all likelihood quotients $L_k(t_i)$ from Eq. (13) are large and close in magnitude, then no clear indication of the true parameter's value is provided, and it is more appropriate to expand the bank than to attempt to move it. The error covariance could then be monitored to make the decision to return it to the smaller size. Since Eq. (16) depends upon the current choice of a_k values, this error covariance is not a reliable indicator for the decision to expand; it is artificially bounded from above by the current size of the bank.

Allowing for precomputability, each filter is an implementation of Eqs. (8) and (10), with the appropriate values for $\Phi_k(t_{i+1}, t_i)$, $B_k(t_i)$, and $K_k(t_i)$ stored for each a_k. When the bank is either moved or expanded, any filters corresponding to newly declared a_k locations must be initialized with $\hat{x}_k(t_i)$ and $p_k(t_i)$ values. A reasonable choice for the $\hat{x}_k(t_i)$'s is the current moving-bank estimate $\hat{x}(t_i^+)$. For the $p_k(t_i)$'s, one option is to set all (those corresponding to newly declared a_k's and those associated with filters that have been retained in the bank during the move) to $1/K$. However, this may result in sluggish convergence to a good parameter estimate. Another choice is to reset only the $p_k(t_i)$'s for filters with newly declared a_k values: the total probability weight of one minus the sum of the unreset $p_k(t_i)$'s is divided among the new filters in the bank. Although this can be apportioned equally, better performance results when it is divided in a manner to reflect an estimate of relative correctness of a_k values, i.e., dividing it proportionately to the evaluation of Eq. (12) for each new a_k.

Before a move or expansion, it may be desirable to "warm up" the new filters before bringing them on line, allowing initial transients in $\hat{x}_k(t_i)$ and $p_k(t_i)$ to die out. To accomplish this, the "move" threshold is left intact, but an additional, tighter "warm up" threshold is used to indicate whether or not to warm up any filters. Until the second threshold is surpassed, these new elemental filters do not affect the adaptive filter output.

Adaptive Control

Three different "assumed certainty equivalence design"[7] approaches can be used to synthesize an adaptive controller, based on the estimator of the previous sections. Using the system model of Eqs. (4) and (5) and an appropriately chosen quadratic cost function, a standard *LQ* regulator can be designed for each a_k value as

$$u_k(t_i) = -G_c^*(t_i; a_k)x_k(t_i) \tag{17}$$

where the optimal controller gain is generated by solving a backward Riccati difference equation. One form of adaptive control law can be implemented as

$$u(t_i) = -G_c^*[t_i; \hat{a}(t_{i-1})]\hat{x}(t_i^+) \tag{18}$$

using the adaptive state estimate $\hat{x}(t_i^+)$ and the parameter estimate provided by Eq. (15) to evaluate the precomputed function $G_c^*(t_i; \cdot)$. Here, $\hat{a}(t_{i-1})$ is used rather than $\hat{a}(t_i)$ to reduce computational delay.

A closely associated control law is given by

$$u(t_i) = -G_c^*(t_i; a_{\text{nom}})\hat{x}(t_i^+) \tag{19}$$

where the controller gain is evaluated on the basis of a nominal parameter value a_{nom}, chosen to adequately provide robust control for all possible parameter values. In this law, the adaptive nature of the estimate is exploited only to enhance the accuracy of the state estimate, not to adjust controller gains in real time.

A third approach is multiple model adaptive control (MMAC). A separate elemental control law of the form

$$u_k(t_i) = -G_c^*(t_i; a_k)\hat{x}_k(t_i^+) \tag{20}$$

is associated with each a_k value and thus with each elemental filter in the bank of K filters. The adaptive control is then generated as the probability-weighted average

$$u(t_i) = \sum_{k=1}^{K} u_k(t_i) \cdot p_k(t_i) \tag{21}$$

Performance Analysis

A 100-run Monte Carlo analysis was used to evaluate the performance potential of a moving-bank estimator and controller, relative to a benchmark of a single Kalman filter, or linear-quadratic-Gaussian controller, based on artificial, perfect knowledge of the "true" parameter value.[33] A simple but physically motivated example was chosen as a single-input/single-output system, described by a damped second order model with two uncertain parameters, damping ratio ζ, and undamped natural frequency ω_n. This corresponds, for example, to a bending mode in an aerospace vehicle model. This continuous-time second order model was subjected to white dynamics driving noise of unit strength; and an equivalent discrete-time model of the form of Eqs. (4) and (5) was generated for extracting sampled data measurements of position every 0.01 s, with $H = [1\ 0]$ in a phase variable state model. The measurement noise variance associated with Eq. (5) was $R = 0.01$. The damping ratio ζ was assumed to lie between 0 and 1, and ω_n could assume values between 2π and 20π rad/s. The "truth model" for simulation and the model upon which each elemental filter was based were identical except for the values ζ and ω_n.

The parameters were each discretized into ten values, forming a 10×10 grid in the two-dimensional parameter space; ζ was linearly discretized over its range, while ω_n was logarithmically discretized. Thus, a full-bank multiple model adaptive estimator would be composed of 100 elemental filters, each based on a model of the form of Eqs. (4) and (5). By comparison, the moving-bank estimator was composed of nine elemental filters corresponding to a 3×3 array of points within the full grid. For all algorithms evaluated, only steady state constant-gain elemental filters and controller gains were considered.

The purpose of this investigation was to assess the effectiveness of a moving-bank estimator or controller, using any of the decision logics of "Moving the Bank", under realistic conditions on the "true" parameter:

1) is constant and equal to one of the discretized values; it may lie outside the initial location of the 3×3 moving bank; points were chosen to cover the parameter space, at $\zeta - \omega_n$ grid indices (1,3), (2,9), (5,4), (9,2), and (10,10);

2) is constant, not equal to one of the discretized values; ($\zeta = 0.07$, $\omega_n = 9$) and ($\zeta = 0.93$, $\omega_n = 41$);

3) slowly varies in a continuous manner; a linear variation between the values in 2) was used;

4) undergoes a jump in value; a jump between the values in 2) was used.

The ability of the moving-bank estimator to provide adequate state estimation accuracy is the primary criterion of performance: our main objective is to design a good adaptive state estimator, not a parameter identifier. Thus, plots of state estimation error statistics are used to compare the capabilities of the various decision logics. The primary figure of merit is acquisition time, the time at which the state error statistics of the moving-bank estimator match those of a benchmark (a single Kalman filter with artificial knowledge of the true parameters) to four significant figures. However, for additional insights, error statistics on parameter estimates were also generated.

Table 1 Thresholds for decision logic

Move decision logic	Move	Warm-up
Residual monitoring	7	3
Parameter position estimate monitoring	0.04	0.02
Parameter position and velocity estimate monitoring	0.04	0.02
Probability monitoring	0.05	0
Bank size change logic	**Change**	**Warm-up**
Covariance monitoring coarse → intermediate	0.08	0.10
Covariance monitoring intermediate → fine	0.0375	0.05
Residual monitoring expand to coarsest	25	Not used

First, the thresholds in each of the decision logics required setting. A single threshold was chosen to allow each logic to provide adequate performance under all conditions on the true parameter. Table 1 presents the thresholds used for both "move" and "warm up" decisions. Consider, for example, the residual monitoring "move" threshold of 7. If that threshold was set too high, the moving bank took longer to identify the system parameters; if it was set too low, the moving bank failed to maintain "lock" on the true parameters. It should be noted that the thresholds have a more pronounced effect on where the bank is centered than on where $\hat{a}(t_i)$ is located, due to its computation via Eq. (15): some continuity of the $\hat{a}(t_i)$ calculation is maintained during a bank move. For probability monitoring, the threshold was set so low that the bank essentially moved whenever the probability weightings on the perimeter bank members exceeded the weighting on the center filter. For the "warm up" thresholds, a value that was too low would cause the wrong filters to be warmed up consistently (the move decision later would be in a different direction), while a value that was too close to the "move" threshold would cause the two decisions to be made simultaneously. In either case there would be effectively no warm up.

Also in Table 1 are the thresholds for the logic to contract and expand the size of the bank. From an initial coarse discretization of the entire admissible parameter space, the bank is allowed to contract to an intermediate and then to the finest discretization, based on a scalar function of $P_a(t_i)$ formed so that equal distances on the two-dimensional grid of parameter points contribute equally to this function.[33] As explained earlier, residual monitoring was used for the decision to expand the bank rather than move it, as to adapt to jumps in parameter values.

Figures 2-5 are plots of the error mean $\pm$ one standard deviation time histories in estimating the velocity state (estimating position is easier since we have direct measurements of position), using the four "move" decision logics of the section entitled "Moving the Bank." In each figure, the middle trace corresponds to the mean error committed by the adaptive estimator, and the two curves symmetrically displaced about it are the mean $\pm$ one error standard deviation. These plots pertain to the case of the true parameter value undergoing the significant jump from (0.07, 9) to (0.93, 41), at 4 s into the simulation. Prior to that time, the exhibited performance is that seen for a true parameter value of (0.07, 9); then there is quick convergence to that of the second $\boldsymbol{a}$ value. The oscillations that are evident, particularly in the first four seconds, are due to the fact that neither parameter value corresponds to a discrete value on the grid in parameter space; the bank tends to oscillate, affecting the state estimates. These oscillations are accentuated by the use of a 20 Hz dither control of amplitude 25 (large compared to the dynamics noise signal) that is put into the system to enhance identifiability and differences between good and bad estimation. (Without it, the system is not excited enough to distinguish between all the algorithms that estimate a nearly quiescent state; feedback control would also serve this function.) As anticipated, residual monitoring is less able to maintain good estimation accuracy than parameter position estimate or probability monitoring. The attempt to put lead into the parameter estimate through position and velocity estimate monitoring in fact adversely affected the stability, which is to be expected.

Table 2 summarizes the performance evaluations in terms of the acquisition time defined earlier. The best overall behavior was achieved by the parameter position estimate monitoring and probability monitoring. Residual monitoring consistently took longer to acquire good state and parameter estimates; in two cases it completely failed to identify the parameters. Parameter position and velocity monitoring had no faster acquisition time than did parameter position monitoring. Moreover, it exhibited the additional undesirable characteristic of "losing lock" once the parameters had been identified. In all of these cases, the small (finely discretized)

Table 2 Acquisition times, s

Mode	Parameter value						
	1	2	3	4	5	6	7
1	1	1.06	0.09	5.25	—[a]	1.38	—[a]
2	0.56	1.06	0.09	2.63	3.63	1.38	3.63
3	0.85[b]	1.06	0.13[b]	0.88[b]	3.63	—[a]	3.88
4	0.53[b]	1.06	0.47	1.25	3.63	1.38	3.63
5	1.06[b]	0.63	1.25	3.25	2.25	1.38	0.25

	Mode		Parameter value
1	Residual monitoring	1	(0,10.5) [grid point (1,3)]
2	Parameter position estimate monitoring	2	(0.11,48.7) [grid point (2,9)]
		3	(0.44,13.5) [grid point (5,4)]
3	Parameter position and velocity estimate monitoring	4	(0.89,8.12) [grid point (9,2)]
4	Probability monitoring	5	(1.0,62.8) [grid point (10,10)]
5	Probability monitoring with acquisition cycle	6	(0.07,9)
		7	(0.93,41)

[a]Failed to identify. [b]Lost lock.

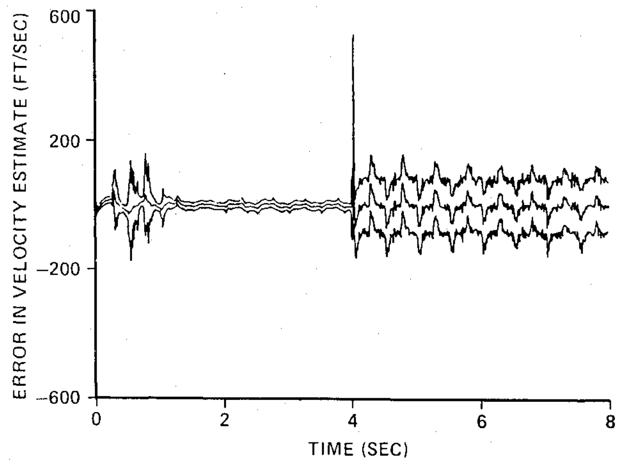

Fig. 2 Velocity error mean $\pm$ 1 standard deviation: residual monitoring.

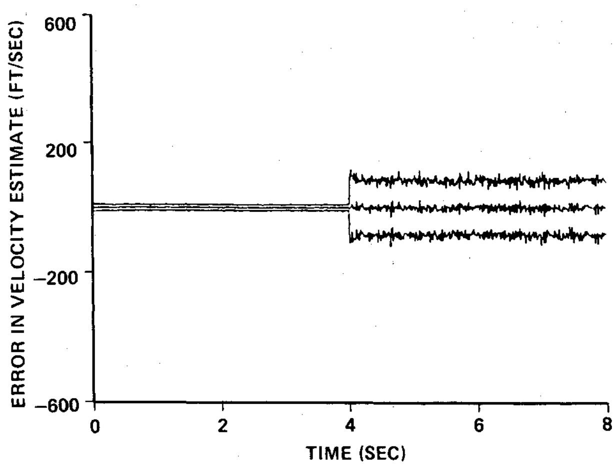

Fig. 3 Velocity error mean $\pm$ 1 standard deviation: parameter position estimate monitoring.

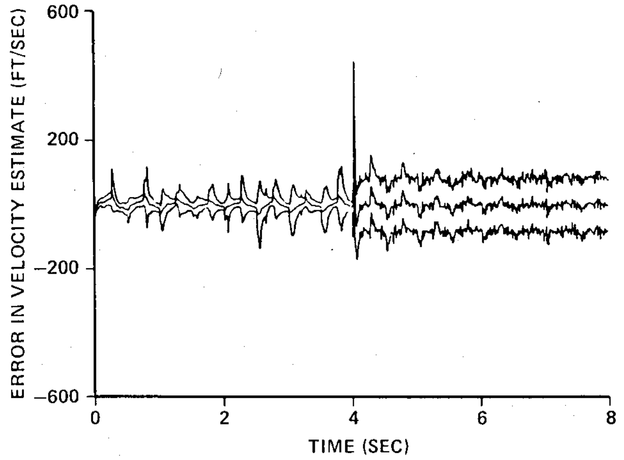

Fig. 4 Velocity error mean ± 1 standard deviation: parameter position and velocity estimate monitoring.

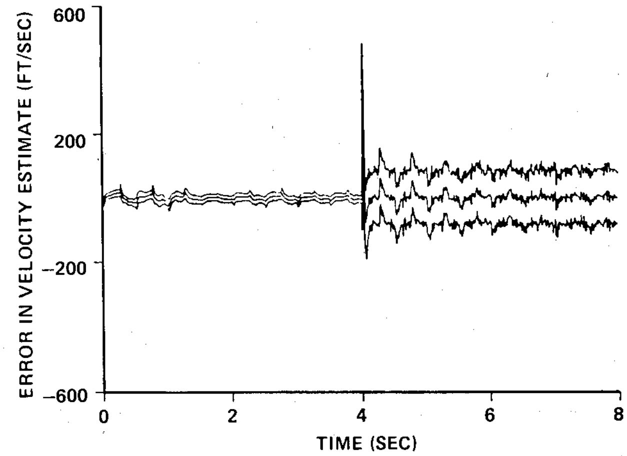

Fig. 5 Velocity error mean ± 1 standard deviation: probability monitoring.

bank was started in the middle of the grid in parameter space. Thus, in all but parameter case 3, it was forced to move to a parameter location outside its initial perimeter. In comparison, the algorithm that used a covariance monitoring acquisition cycle performed quite well in some instances (high true natural frequency), but failed in others (low true natural frequency).

For all of the decision methods, filter "warm up" was found to have little, if any, effect on performance. In view of this and the additional elemental filters and computation time needed, filter "warm up" is not warranted.

In all cases, the moving-bank algorithm had less difficulty identifying ω_n than ζ. This could be heuristically predicted by considering their relative effects on the power spectral density of the position process that is sampled through the measuring device. This was corroborated by a more rigorous generalized ambiguity function analysis,[4,33] which indicated there is considerably more information available through these measurements about ω_n than ζ.

Probability monitoring was used as a decision logic for moving the bank in conjunction with adaptive controller synthesis. In all cases, the control was disabled for the first second of the simulation. If the control is applied before the parameters have been identified (especially ω_n), the wrong control is applied, often driving the system unstable. An explicit caution factor as a function of $P_a(t_i)$ can be used with the control, rather than using the ad hoc period of 1 s, to ensure that wrong control does not destabilize the system.

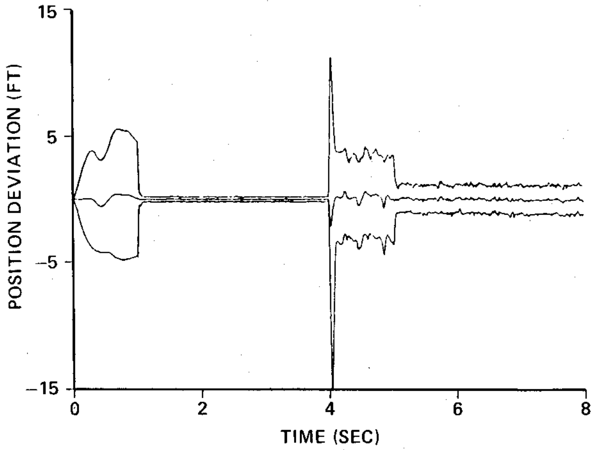

Fig. 6 Position state mean ± 1 standard deviation MMAE-based control; true *a* undergoes jump.

More is presented about this in the later discussion of control law [Eq. (19)]. The controller gains were designed on the basis of quadratic weighting terms of 10^5 on state deviations, 10 on velocity, and 10^{-2} on control; these were purposely chosen to provide "tight" control of position if parameters are appropriately identified. Figure 6 presents the time history of statistics for the position state process resulting from the use of control law (18) for the case of the true parameter undergoing the jump associated with the previous figures (the dither signal was removed once feedback control was applied). During the first second, no control was applied. Similarly, control was disenabled to allow reacquisition after the jump; this decision was made by residual monitoring, with a threshold of 25, as used in Table 1 for reacquisition decisions. During the periods when the feedback control was applied, the performance was indistinguishable from that of the benchmark LQG controller with artificial knowledge of the true parameters. MMAC control generated as in Eqs. (20) and (21) yielded virtually identical results. This replication of benchmark results was also achieved for all other conditions on the true parameter besides the jump condition.

In determining the nominal parameter value for the control law, (19) it was found that the controller was insensitive to true ζ variations but highly sensitive to ω_n. When the true ω_n was higher than the nominal assumed in the controller, the closed loop system became unstable. For example, for an assumed value of 17.48 rad/s in the controller, all true frequencies above 18.61 rad/s produced unstable performance. Therefore, the highest possible ω_n value and the midrange ζ were chosen. When true ω_n was high, this controller performed adequately, with mean squared state values on the order of 1.5 times the benchmark results; this degraded to 15 times the benchmark values at lower true ω_n's. This type of control would be useful to consider for reacquisition times, as seen at the 4 s point in Fig. 6, rather than disenabling control altogether.

Conclusion

The feasibility and performance capabilities of moving-bank multiple model adaptive estimation and control algorithms have been investigated. In a simple but physically motivated example, they achieved performance essentially equivalent to a benchmark Kalman filter or linear-quadratic-Gaussian controller, with artificial knowledge of true parameter values, after a very short transient. Of the decision logics explored for moving the bank, parameter position estimate monitoring and probability monitoring provide the best performance. This performance potential and the significant reduction in computational loading compared to full-bank multiple model adaptive estimation and control algorithms are such that continued development is warranted.

References

[1]Magill, D.T., "Optimal Adaptive Estimation of Sampled Stochastic Processes," *IEEE Transactions on Automatic Control*, Vol. AC-10, No. 5, Oct. 1965, pp. 434-439.

[2]Athans, M. and Chang, C.B., "Adaptive Estimation and Parameter Identification Using Multiple Model Estimation Algorithm," Technical Note 1976-28, ESD-TR-76-184, Lincoln Laboratory, Lexington, MA, June 1976.

[3]Lainiotis, D.G., "Partitioning: A Unifying Framework for Adaptive Systems, I: Estimation," *Proceedings of the IEEE*, Vol. 64, Aug. 1976, pp. 1126-1143.

[4]Maybeck, P.S., *Stochastic Models, Estimation and Control*, Vol. 2, Academic Press, New York, 1982.

[5]Meer, D.E., "Multiple Model Adaptive Estimation for Space-Time Point Process Observations," Ph.D. dissertation, Air Force Institute of Technology, Wright-Patterson AFB, OH, Sept. 1982.

[6]Meer, D.E. and Maybeck, P.S., "Multiple Model Adaptive Estimation for Space-Time Point Process Observations," *Proceedings of the IEEE Conference on Decision & Control*, Las Vegas, NV, Dec. 1984, pp. 811-818.

[7]Maybeck, P.S., *Stochastic Models, Estimation and Control*, Vol. 3, Academic Press, New York, 1982.

[8]Maybeck, P.S. and Zicker, W.L., "MMAE-Based Control for Space-Time Point Process Observations," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-21, No. 3, May 1985, pp. 292-300.

[9]Athans, M., et al., "The Stochastic Control of the F-8C Aircraft Using a Multiple Model Adaptive Control (MMAC) Method—Part 1: Equilibrium Flight," *IEEE Transactions on Automatic Control*, Vol. AC-22, No. 5, Oct. 1977, pp. 768-780.

[10]Hawkes, R.M. and Moore, J.B., "Performance Bounds for Adaptive Estimation," *Proceedings of the IEEE*, Vol. 64, Aug. 1976, pp. 1143-1150.

[11]Tugnait, J.K., "Convergence Analysis of Partitioned Adaptive Estimators Under Continuous Parameter Uncertainty," *IEEE Transactions on Automatic Control*, Vol. AC-25, No. 3, June 1980, pp. 569-573.

[12]Dasgupta, S. and Westphal, L.C., "Convergence of Partitioned Adaptive Filters for Systems with Unknown Biases," *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 5, May 1983, pp. 614-615.

[13]Chang, C.B. and Athans, M., "State Estimation for Discrete Systems with Switching Parameters," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-14, No. 3, May 1978, pp. 418-424.

[14]Moose, R.L. and Wang, P.P., "An Adaptive Estimator with Learning for a Plant Containing Semi-Markov Switching Parameters," *IEEE Transactions on Systems, Man, and Cybernetics*, May 1973, pp. 277-281.

[15]Weiss, J.L., Upadhyay, T.N., and Tenney, R.R., "Finite Computable Filters for Linear Systems Subject to Time Varying Model Uncertainty," *Proceedings of the IEEE National Aerospace and Electronics Conference*, Dayton, OH, May 1983, pp. 349-355.

[16]Fry, C.M. and Sage, A.P., "On Hierarchical Structure Adaptation and Systems Identification," *International Journal of Control*, Vol. 20, No. 3, 1974, pp. 433-452.

[17]Lamb, P.R. and Westphal, L.C., "Simplex-Directed Partitioned Adaptive Filters," *International Journal of Control*, Vol. 30, 1979, pp. 617-627.

[18]Thorp, J.S., "Optimal Tracking of Maneuvering Targets," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-9, No. 4, July 1973, pp. 512-519.

[19]Moose, R.L., "An Adaptive State Estimation Solution to the Maneuvering Target Problem," *IEEE Transactions on Automatic Control*, Vol. AC-20, No. 3, June 1975, pp. 359-362.

[20]Tenney, R.R., Hebbert, R.S., and Sandell, N.R. Jr., "A Tracking Filter for Maneuvering Sources," *IEEE Transactions on Automatic Control*, Vol. AC-22, No. 2, Mar. 1977, pp. 246-261.

[21]Gholson, N.H. and Moose, R.L., "Maneuvering Target Tracking Using Adaptive State Estimation," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-13, No. 3, May 1977, pp. 310-317.

[22]Moose, R.L., Van Landingham, H.F., and McCabe, D.H., "Modeling and Estimation for Tracking Maneuvering Targets," *IEEE Transactions on Aeorspace and Electronic Systems*, Vol. AES-15, No. 3, May 1979, pp. 448-456.

[23]Korn, J. and Beean, L., "Application of Multiple Model Adaptive Estimation Algorithms to Maneuver Detection and Estimation," Tech. Rept. TR-152, Alphatech, Inc., Burlington, MA, June 1983.

[24]Suizu, R.I., "Enhanced Tracking of Airborne Targets Using Multiple Model Filtering Techniques for Adaptive Field of View Expansion," M.S. thesis, Air Force Institute of Technology, Wright-Patterson AFB, OH, Dec. 1983.

[25]Chang, C.B. and Tabaczynski, J.A., "Application of State Estimation to Target Tracking," *IEEE Transactions on Automatic Control*, Vol. AC-29, No. 2, Feb. 1984, pp. 98-109.

[26]Maybeck, P.S. and Suizu, R.I., "Adaptive Tracker Field-of-View Variation Via Multiple Model Filtering ," *IEEE Transactions on Aerospace and Electronic Systems*,Vol. AES-21, No. 4, July 1985, pp. 529-539.

[27]Brown, R.G., "A New Look at the Magill Adaptive Estimator as a Practical Means of Multiple Hypothesis Testing," *IEEE Transactions on Circuits and Systems*, Vol. CAS-30, No. 10, Oct. 1983, pp. 765-768.

[28]Willsky, A.S., et al., "Dynamic Model-Based Techniques for the Detection of Incidents on Freeways," *IEEE Transactions on Automatic Control*, Vol. AC-25, No. 3, June 1980, pp. 347-359.

[29]Sims, C.S. and D'Mello, M.R., "Adaptive Deconvolution of Seismic Signals," *IEEE Transactions on Geoscience Electronics*, Vol. GE-16, April 1978, pp. 99-103.

[30]Hostetler, L.D. and Andreas, R.D., "Nonlinear Kalman Filtering Techniques for Terrain-Aided Navigation," *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 3, March 1983, pp. 315-323.

[31]Mealy, G.L. and Tang, W., "Application of Multiple Model Estimation to a Recursive Terrain Height Correlation System," *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 3, March 1983, pp. 323-331.

[32]Maybeck, P.S., "Practical Adaptive Estimation (and Control) Algorithms," unpublished notes, Air Force Institute of Technology, Wright-Patterson AFB, OH, Oct. 1979, updated Oct. 1983.

[33]Hentz, K.P., "Feasibility Analysis of Moving Bank Multiple Model Adaptive Estimation and Control Algorithms," M.S. thesis, Air Force Institute of Technology, Wright-Patterson AFB, OH, Dec. 1984.

[34]Baram, Y., "Information, Consistent Estimation and Dynamic System Identification," Electronic Systems Laboratory, Department of Electrical Engineering, MIT, Cambridge, MA, Rept. ESL-R-718, Nov. 1976.

[35]Baram, Y. and Sandell, N.R. Jr., "An Information Theoretic Approach to Dynamic System Modeling and Identification," *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 1, Jan. 1978, pp. 61-66.

[36]Baram, Y. and Sandell, N.R. Jr., "Consistent Estimation of Finite Parameter Sets with Application to Linear Systems Identification," *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 3, June 1978, pp. 451-454.

[37]Yared, K.I., "On Maximum Likelihood Identification of Linear State Space Models," Ph.D. dissertation, MIT Laboratory for Information and Decision Systems, Cambridge, MA, Rept. LIDS-TH-920, July 1979.